ESERCIZI SUI LIMITI

$$\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\log_a e \Rightarrow \lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

$$\lim_{x\to 0}\frac{\log_3(1+x^2)}{x}=0 \qquad \boxed{\frac{0}{0}}$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \Rightarrow \lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$\lim_{x\to 0}\frac{2^{3x}-1}{5x}=\frac{3}{5}\ln 2 \qquad \boxed{\frac{0}{0}}$$

$$\frac{(1+x)^a-1}{x}=\alpha\ ;\ \lim_{x\to 0}\frac{\sqrt{1+4x}-1}{x}=2 \quad \boxed{\frac{0}{0}}$$

$$\lim_{n\to+\infty}\left(\frac{n}{n+2}\right)^n=\dots$$

$$\lim_{n\to+\infty}\left(\frac{n}{n+2}\right)^n \sim \left(\frac{n}{n}\right)^n \sim 1^\infty \quad \text{F. I.}$$

MODO 1

$$\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^x$$

$$\lim_{x\to\infty}\left(\frac{x+2-2}{x+2}\right)^x$$

$$\lim_{x\to\infty}\left(1-\frac{2}{x+2}\right)^x$$

$$\lim_{x \to \infty} \left(1 - \frac{2}{x+2}\right)$$

$$\lim_{t \to \infty} \left(1 + \frac{1}{t}\right)^t = e$$

$$\lim_{x \to \infty} \left[\left(1 - \frac{2}{x+2}\right)^{\frac{-\frac{x+2}{2}}{-\frac{x+2}{2}}}\right]^x$$

$$= \lim_{x \to \infty} \left[\underbrace{\left(1 - \frac{2}{x+2}\right)^{-\frac{x+2}{2}}}_{= e}\right]^{-\frac{2}{x+2} \cdot x}$$

$$= \frac{-2x}{x+2} \sim \frac{-2x}{x} \sim -2$$

$$= e^{-2}$$

$$\lim_{n \to +\infty} \left(\frac{n}{n+2}\right)^n = \dots$$

$$K = e^{\ln K}$$

$$\lim_{n \to +\infty} e^{\ln \left(\frac{n}{n+2}\right)^n}$$

$$= \lim_{n \to +\infty} e^{n \cdot \ln\left(\frac{n}{n+2}\right)}$$

POICHÉ $\frac{n}{n+2} \to 1 \to \ln \frac{n}{n+2} \to 0$

Ricordiamo il limite notevole

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1 \to \ln(1+x) \sim x$$

$$t = 1 + x \to x = t - 1$$

$$\ln t \to t - 1$$

$$\log \frac{n}{n+2} \sim \frac{n}{n+2} - 1$$

Riprendo il limite

$$= \lim_{n \to +\infty} e^{n \cdot \ln\left(\frac{n}{n+2}\right)}$$

$$= \lim_{n \to +\infty} e^{n \cdot \left(\frac{n}{n+2} - 1\right)}$$

$$= \lim_{n \to +\infty} e^{n \cdot \frac{\cancel{n} - \cancel{n} - 2}{n+2}}$$

$$= \lim_{n \to +\infty} e^{\frac{-2n}{n+2}} \sim \frac{-2n}{n} \sim -2$$

$$= e^{-2} = \frac{1}{e^2}$$

$$\lim_{n\to+\infty} \frac{n-n^2}{1+10n\log n} = \dots$$

$$\lim_{n\to+\infty} \frac{n-n^2}{1+10\,n\cdot\log n} = \frac{\infty}{\infty} = \text{F. I.}$$

SCALA ∞ $\quad n = o(n^2)$

$$= \lim_{n\to+\infty} \frac{-n^{\cancel{2}}}{10\,\cancel{n}\cdot\log n}$$

$$-\frac{n}{10\log n} \sim -\infty \qquad \log n = o(n)$$

$$\lim_{n\to+\infty} n\left(\log(n+1)-\log n\right) = \dots$$

$$\lim_{n \to +\infty} n \cdot \left(\log(n+1) - \log n \right) = \infty \cdot (\infty - \infty) = \text{F.I.}$$

$$= \lim_{n \to +\infty} n \cdot \log \frac{n+1}{n}$$

$$\log \frac{n+1}{n} \longrightarrow \frac{n+1}{n} - 1$$

$$= \lim_{n \to +\infty} n \cdot \left(\frac{n+1}{n} - 1 \right)$$

$$= \lim_{n \to +\infty} n \cdot \frac{1}{n} = 1$$

$$\lim_{n \to +\infty} n \left(\log^3(n+1) - \log^3 n \right) = \ldots$$

$$\lim_{n \to +\infty} n \cdot \left(\log^3(n+1) - \log^3 n \right)$$

$$= \infty \, (\infty - \infty)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$\log^3(n+1) - \log^3 n$$

$$= \left(\log(n+1) - \log n\right) \cdot \left[\log^2(n+1) + \log(n+1)\cdot\log n + \log^2 n\right]$$

$$\downarrow$$

$$\log\frac{n+1}{n}$$

$$\lim_{n\to+\infty} n \cdot \left(\log^3(n+1) - \log^3 n\right)$$

$$\lim_{n\to\infty} \underbrace{n \cdot \log\frac{n+1}{n}} \cdot \left[\log^2(n+1) + \log(n+1)\cdot\log n + \log^2 n\right]$$

$$\sim 3\log^2 n$$

$$\sim n \cdot \left(\frac{n+1}{n} - 1\right)$$

$$\sim n\,\frac{1}{n} \sim 1$$

$$\lim_{n\to+\infty} 3\log^2 n = +\infty$$

$$\lim_{n\to+\infty} e^n\left(n - \log\left(e^n + n\right)\right) = \dots$$

$$\lim_{x\to+\infty} e^n\left(n - \log\left(e^n + n\right)\right) = e^{+\infty}\cdot(\infty - \infty) \quad \text{F.I.}$$

$$\lim_{x \to +\infty} e^{n} \left(n - \log \left(e^{n} \cdot \left(1 + \frac{n}{e^{n}} \right) \right) \right)$$

$$\lim_{n \to +\infty} e^{n} \left(n - \left(\log e^{n} + \log \left(1 + \frac{n}{e^{n}} \right) \right) \right)$$

$$\lim_{n \to +\infty} e^{n} \left(\cancel{n} - \cancel{n} - \log \left(1 + \frac{n}{e^{n}} \right) \right)$$

$$\lim_{n \to +\infty} - e^{n} \cdot \log \left(1 + \frac{n}{e^{n}} \right)$$

TENDE A ZERO

$$\log \left(1 + \frac{n}{e^{n}} \right) \sim \frac{n}{e^{n}}$$

$$\lim_{n \to +\infty} - \cancel{e^{n}} \cdot \frac{n}{\cancel{e^{n}}}$$

$$\lim_{n \to +\infty} - n = - \infty$$

$$\lim_{n \to +\infty} \sqrt[n]{e^n + 1} = \ldots$$

$$\lim_{n \to +\infty} \sqrt[n]{e^{n} + 1} = \sqrt[\infty]{\infty} = \infty^{1/\infty} = \infty^{0} \quad \text{F.I.}$$

$$e^{n} + 1 \to e^{n}$$

$$\sim \sqrt[n]{e^{n}} = e$$