SUCCESSIONI CONVERGENTI, DIVERGENTI E IRREGOLARI

Studiare il carattere delle seguenti successioni:

1) $$a_n = \frac{2^{-n} + 3n^3}{\log^6 n + 2 - n^6}$$

$$\lim_{n \to +\infty} \frac{2^{-n} + 3n^3}{\log^6 n + 2 - n^6} = \frac{2^{-\infty} + \infty^3}{\log^6(\infty) + 2 - \infty^6} = \frac{\infty}{\infty}$$

Nota bene! $2^{-\infty} \longrightarrow 0$

$$\frac{2^{-n} + 3n^3}{\log^6 n + 2 - n^6} \sim \frac{3n^3}{-n^6} = -\frac{3}{n^3} \longrightarrow 0$$

La successione è CONVERGENTE!

2) $$a_n = \frac{\log n + 3n^3 \log n}{2^{1/n} + n^5}$$

$$\lim \frac{\log n + 3n^3 \log n}{\quad} = \frac{\infty}{\quad}$$

$$n \to +\infty \quad 2^{1/n} + n^5 \quad \infty$$

Nota bene! $2^{1/\infty} \to 2^0 = 1$

$$\frac{\log n + 3n^3 \log n}{2^{1/n} + n^5} \sim$$

$$\frac{3n^3 \cdot \log n}{n^5} = \frac{3 \log n}{n^2} \to 0 \qquad n^2 > \log n$$

La successione è CONVERGENTE

$$a_n = \frac{n^{2+\frac{1}{n}}}{1 + n^{3-\frac{2}{n}}}$$

$$\lim_{n \to +\infty} \frac{n^{2+\frac{1}{n}}}{1 + n^{3-\frac{2}{n}}} = \frac{\infty^{2+\frac{1}{\infty}}}{1 + \infty^{3-\frac{2}{\infty}}} = \frac{\infty}{\infty}$$

Nota bene! $\frac{1}{n} \to 0 \quad \text{se } n \to \infty$

$-\frac{2}{n} \to 0 \quad \text{se } n \to \infty$

$$\frac{n^{2+\frac{1}{n}}}{1 + n^{3-\frac{2}{n}}} \sim \frac{n^2}{n^3} = \frac{1}{n} \to 0$$

La successione è CONVERGENTE

4) $\lim_{n \to +\infty} \left(\frac{n}{n+1} \right)^{2n} = \left(\frac{\infty}{\infty} \right)^{\infty}$

$$\left(\frac{n}{n+1} \right)^{2n} \sim \left(\frac{n}{n} \right)^{2n} = 1^{2n} \to 1^{\infty} \quad \text{F.I.}$$

$$\lim_{n \to +\infty} \left(\frac{n}{n+1} \right)^{2n}$$

$$= \lim_{n \to +\infty} e^{2n \cdot \log\left(\frac{n}{n+1}\right)}$$

Da notare che: se $n \to \infty$

$$\log \frac{n}{n+1} \to 0 \quad \to$$

$$\log \frac{n}{n+1} \sim \frac{n}{n+1} - 1 = \frac{n-(n+1)}{n+1}$$

$$= \lim_{n \to +\infty} e^{2n \cdot \frac{-1}{n+1}} \qquad \frac{-2n}{n+1} \sim \frac{-2n}{n} = -2$$

$$= e^{-2}$$

Dunque la successione CONVERGE!

5) $a_n = \left(\frac{n+2}{\quad} \right)^{n^2}$

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$$\lim_{n \to +\infty} \left(\frac{n+2}{n+1}\right)^{n^2} = \left(\frac{\infty}{\infty}\right)^{\infty}$$

$$\left(\frac{n+2}{n+1}\right)^{n^2} \sim \left(\frac{n}{n}\right)^{n^2} = 1^{\infty} \quad \text{se } n \to \infty$$

$$\lim_{n \to +\infty} \left(\frac{n+2}{n+1}\right)^{n^2}$$

$$= \lim_{n \to +\infty} e^{n^2 \cdot \log\left(\frac{n+2}{n+1}\right)}$$

$$\log\left(\frac{n+2}{n+1}\right) \sim \log \frac{n}{n} = \log 1 = 0$$

$$\log\left(\frac{n+2}{n+1}\right) \sim \frac{n+2}{n+1} - 1 = \frac{1}{n+1} \quad \text{se } n \to \infty$$

$$= \lim_{n \to +\infty} e^{n^2 \cdot \frac{1}{n+1}} \qquad \frac{n^2}{n+1} \sim \frac{n^2}{n} = n$$

$$\sim \lim_{n \to +\infty} e^{n} = e^{+\infty} = +\infty$$

La successione DIVERGE al più infinito!

6) $$a_n = \left(1 + \frac{3}{n}\right)^{-n}$$

$$\lim_{n \to +\infty} \left(1 + \frac{3}{n}\right)^{-n} = \left(1 + \frac{3}{\infty}\right)^{-\infty} = 1^{-\infty} \quad \text{F.I.}$$

$$= \lim_{n \to \infty} \left[\left(1 + \frac{3}{n}\right)^{\frac{n}{3}}\right]^{-3} = e^{-3} \qquad \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

La successione è CONVERGENTE