

1-INDICI01- media varianza correlazione

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7 The following table contains 10 scores of a variable. Obviously there is an outlier which is gray-colored. What will happen if the outlier is ignored?

25	25	26	24	2	29	25	22	27	25
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- (a) The mean and the median will increase notably.
- (b) The mean will increase notably whereas the median remains the same.
- (c) The median will increase notably whereas the mean remains the same.
- (d) Both the mean and median remain unchanged.

MEDIANA

ORDINIAMO I DATI

POSIZIONE	1	2	3	4	5	6	7	8	9	10
VALORI	2	22	24	25	25	25	25	26	27	29

$$n = 10 \text{ (PARI)} \rightarrow 2 \text{ POSIZIONI}$$
$$\frac{n}{2} = 5 \quad \frac{n}{2} + 1 = 6$$

POSIZIONE	1	2	3	4	5	6	7	8	9	10
VALORI	2	22	24	25	25	25	25	26	27	29

MEDIANA = 25

MEDIA

POSIZIONE	1	2	3	4	5	6	7	8	9	10
VALORI	2	22	24	25	25	25	25	26	27	29

$$\bar{x} = \frac{2 + 22 + 24 + 25 \cdot 4 + 26 + 27 + 29}{10} = 23$$

TOGLIAMO IL VALORE ANOMALO

MEDIANA

POSIZIONE	1	2	3	4	5	6	7	8	9
VALORI	22	24	25	25	25	25	26	27	29

$$n = 9 \text{ (DISPARI)} \rightarrow 1 \text{ POSIZIONE} \quad \frac{n+1}{2} = \frac{10}{2} = 5$$

MEDIANA = 25 (rimane invariata)

$$\text{MEDIA} \quad \bar{x} = \frac{22 + 24 + 25 \cdot 4 + 26 + 27 + 29}{9} = 25,33 \text{ (AUMENTA)}$$

18 A discrete random variable X has a mean $\mu = 15$ and a standard deviation $\sigma = 4$. The constant value 8 is added to X in order to create a new variable: $Y = X + 8$. What is the mean of Y ?

- (a) 4
- (b) 15
- (c) 23
- (d) It is not possible to calculate the mean without knowing the probability distribution of X and Y .

$$\begin{aligned} X: \quad \mu_x &= 15 & y &= x + 8 \\ \sigma_x &= 4 & \mu_y &= \mu_{x+8} = \mu_x + 8 = 15 + 8 = 23 \end{aligned}$$

RICORDA:

$$\begin{aligned} X: \quad \mu_x & & y &= ax + b \\ \mu_y &= \mu_{ax+b} = a \cdot \mu_x + b \end{aligned}$$

Deviazione standard

$$\begin{aligned} X: \quad \sigma_x & & y = ax + b: \quad \sigma_y &= \sigma_{ax+b} = a \sigma_x \\ X: \quad \sigma_x &= 4 & y = x + 8 \quad \sigma_y &= \sigma_{x+8} = \sigma_x = 4 \end{aligned}$$

- (a) 4
- (b) 15
- (c) 23
- (d) It is not possible to calculate the mean without knowing the probability distribution of X and Y .

24 Based on 254 observations $\bar{X} = 50$ and $s = 10$ have been calculated. Required is a scale with $\bar{X}_{new} = 50$ and $s_{new} = 5$. To this end, a linear transformation is used where $X_{new} = a + bX$ with $b > 0$. The first observation was 40. What are the coefficients a and b of the required linear transformation and what is the value of the first observation after applying this transformation?

- (a) $a = -50$ and $b = 2$ and the transformed first observation is 30.
- (b) $a = 0$ and $b = 0.5$ and the transformed first observation is 20.
- (c) $a = 0$ and $b = 1$ and the transformed first observation is 40.
- (d) $a = 25$ and $b = 0.5$ and the transformed first observation is 45.

$$n = 254$$

$$\bar{X} = 50 \quad s = 10 \quad x_1 = 40$$

$$\bar{X}_N = 50 \quad s_N = 5 \quad X_N = a + b \cdot x$$

$$\bar{X}_N = a + b \bar{X}$$

$$s_N = b \cdot s$$

$$\begin{cases} 50 = a + 50b \rightarrow a + 50 \cdot 0,5 = 50 \rightarrow a = 25 \\ 5 = b \cdot 10 \rightarrow b = 0,5 \end{cases}$$

$$X_{1N} = a + b \cdot x_1$$

$$X_{1N} = 25 + 0,5 \cdot 40 = 45$$

- (a) $a = -50$ and $b = 2$ and the transformed first observation is 30.
- (b) $a = 0$ and $b = 0.5$ and the transformed first observation is 20.
- (c) $a = 0$ and $b = 1$ and the transformed first observation is 40.
- (d) $a = 25$ and $b = 0.5$ and the transformed first observation is 45.

30 We collected scores of 60 test subjects and know that the variance of X is 4, the variance of Y is 9, and the correlation between X and Y is 0.8. What is the covariance between X and Y ?

- (a) 1.2
- (b) 4.8
- (c) 5.4
- (d) 28.8

$$n = 60$$

$$s_x^2 = 4 \quad s_y^2 = 9 \quad r_{xy} = 0,8 \quad \text{Cov}_{xy} = ?$$

$$r_{xy} = \frac{\text{Cov}_{xy}}{s_x \cdot s_y}$$

$$\begin{aligned} \text{Cov}_{xy} &= \sigma_x \cdot \sigma_y \cdot \rho_{xy} = \\ &= \sqrt{4} \cdot \sqrt{9} \cdot 0,80 = 4,8 \end{aligned}$$

32 It is important that the weight of a figure skater matches the weight of his skating partner. For male figure skaters (X) the average weight is 77 kg with a standard deviation of 4 kg. For female figure skaters (Y) the average weight is 49 kg with a standard deviation of 3 kg. The correlation between X and Y is 0.77. What is the standard deviation of the random variable $W = X - Y$?

- (a) 2.55
- (b) 2.65
- (c) 6.52
- (d) 7.00

$$\begin{aligned} \bar{x} &= 77 & \bar{y} &= 49 & \rho_{xy} &= 0,77 \\ \sigma_x &= 4 & \sigma_y &= 3 & & \end{aligned}$$

$$W = X - Y \quad \sigma_W = ?$$

Variabile somma

$$\begin{aligned} \sigma_W^2 &= \sigma_{X+Y}^2 = \sigma_x^2 + \sigma_y^2 + 2 \cdot \text{Cov}_{xy} \\ &= \sigma_x^2 + \sigma_y^2 + 2 \cdot \rho_{xy} \cdot \sigma_x \sigma_y \end{aligned}$$

Variabile differenza

$$\begin{aligned} \sigma_W^2 &= \sigma_{X-Y}^2 = \sigma_x^2 + \sigma_y^2 - 2 \cdot \text{Cov}_{xy} \\ &= \sigma_x^2 + \sigma_y^2 - 2 \cdot \rho_{xy} \cdot \sigma_x \sigma_y \end{aligned}$$

$$\begin{aligned} \sigma_W &= \sqrt{\sigma_x^2 + \sigma_y^2 - 2 \cdot \rho_{xy} \cdot \sigma_x \sigma_y} \\ &= \sqrt{4^2 + 3^2 - 2 \cdot 0,77 \cdot 4 \cdot 3} = 2,553 \end{aligned}$$

- (a) 2.55
- (b) 2.65
- (c) 6.52
- (d) 7.00